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Remarks on Gauge Theories of Fundamental Forces

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Abstract

By clarifying the concepts of strong and weak gravity in a scalar-scalar-tensor theory of gravitation, we have studied an action principle from which we have discussed several theories of weak and (separately) strong gravity. By further appealing to the general principle of gauge invariance we are able to discuss gauge theories of spontaneously broken discrete symmetries. Finally, we find that super-heavy gauge bosons are automatically excluded when both gravities are properly understood.

1. *Introduction*

Following the initial success of exploiting the gauge principle for the construction of unified theories of electromagnetic and weak interactions (Salam, 1968; Weinberg, 1967), the possibility arises of bringing in the further natural forces, namely, the strong, *CP* violating (if different from the weak force) and, finally, the gravitational interactions. On the other hand, in recent years an attempt has been made to unify the strong and gravitational interactions: this pair of forces was studied in the context of a gravitational field coupled, in Riemannian space, to a second tensor field (strong gravity), in analogy with vector meson dominance (Zumino, 1970; Isham *et al.*, 1971; Aichelburg, 1973). An asymptotic solution was found for the f-field assuming spherical symmetry (Aragone & Chela-Flores, 1972); recently, this solution was used for evaluating the *exact* energy content inside a sphere enclosing a point mass (Chela-Flores, 1974) but, in spite of the fact that good agreement was obtained with the result expected on the grounds of special relativity alone, we felt induced to study a simpler model where *exact* spherically symmetric solutions were known. In other words, Einstein's field equations were adopted without insisting that the classical Newtonion limit be valid (Chela-Flores & Herrera, 1974) (henceforth we shall refer to this theory as C–H).

Although very suggestive as a semi-classical framework in which to describe highly dense hadronic matter, in the C–H model one does not sufficiently use the full power of field theory, even before some of the underlying quantum gravity problems are considered.

We shall extend our earlier considerations to a full theory of gravitation with Weyl-type gauge symmetries (Zumino, 1970). Then, by appealing to the principle of gauge invariance (Weyl, 1950), gauge fields mediating fundamental forces will be introduced; in this way, we are able to go beyond the C-H model. In doing so, we obtain a better understanding of a recent theory of Freund (1974).

The plan of this work is as follows:

In Section 2 we generalise the scalar-tensor theory (Brans & Dicke, 1961) to a scalar-scalar-tensor theory in which, as we proceed to show in Section 3, the Newtonian and Yukawa limits of the full theory set the difference between the two scalar fields.

Then, in Section 4, we consider some special cases of the theory and recover the theory of general relativity (Einstein, 1916), the Brans–Dicke as well as the C-H theory.

Extensions to gauge theories of fundamental forces are considered in Section 5. Particular attention is given to the recent work of Freund, showing from our point of view that we may infer directly the correct order of magnitude for the spontaneously broken *C*-violation in electromagnetism, as well as *CP* violation. We end the discussion of this section by explaining how we naturally avoid the super-heavy vector bosons, which have appeared in the unified theory of strong, electromagnetic and weak interactions studied by Georgi *et al.* (1974).

Finally, a brief summary is given in Section 6.

2. *Unified Theory of Strong and Weak Gravity*

In order to generalise Einstein's theory of gravitation, we may start with the corresponding action principle from which the Euler–Langrange equations of motion are obtained (Landau & Lifshitz, 1951),

$$0 = \delta \int d_4x\sqrt{-g}\left[R + \frac{16\pi}{c^2} G_N L\right]$$

where R is the Riemann scalar and L is the Lagrangian density of matter (including all fields besides gravitation); G_N is the Newtonian coupling constant.

We begin by dividing the Lagrangian density by G_N and assume that *the Machian relation between geometry and matter applies beyond gravitational forces.*

Then, with Brans and Dicke, we replace G_N by a scalar, but choose to write ϕ^2 instead of ϕ as in the original scalar-tensor theory.

In view of the fact that in the unified theory of strong and weak gravity, the strong (i.e. nuclear) potential and the weak (i.e. Newtonian) potential will be *competing* in the relevant long-range and short-range limits, we study a *correspondence principle:*

In problems typical of microphysics, the short-range (Yukawa) potential will dominate the long-range (Newtonian) potential and vice versa, in problems typical of macrophysics, the long-range potentials will be dominant.

Hence, we find it is sufficient to study an *asymptotic expansion* for the gravitational coupling

$$\phi^2 = \Omega^2 + \chi^2 + \ldots \tag{2.1}$$

and we let the physics of the problem identify the leading terms of this expansion (by means of a perturbative treatment).

In order to construct a Lagrangian theory, we consider the functions Ω and χ as *field variables* which couple with the metric tensor $g_{\mu\nu}$ in such a way that the action principle satisfies the Weyl invariance (Zumino, 1970),

$$\Omega \to \bar{\Omega} = \Omega \exp(-\Lambda) \tag{2.2a}$$

$$\chi \to \bar{\chi} = \chi \exp(-\Lambda) \tag{2.2b}$$

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = g_{\mu\nu} \exp(2\Lambda) \tag{2.2c}$$

We are, therefore, led to the simplest generalisation of Einstein's theory embodied in the action principle

$$0 = \delta \int \{\tfrac{1}{6}(\Omega^2 + \chi^2)R + 16\pi L/c^4 - \omega\partial_\mu\Omega\partial g^{\mu\nu} - \xi\partial_\mu\chi\partial_\nu\chi g^{\mu\nu}\} \sqrt{(-g)}\, d_4 x \tag{2.3}$$

It cannot be over-emphasised, at this stage, that the inclusion in a full dynamical theory of coupling strengths that yield results correct only to first order, has an important precedent in the theory of β-decay and other weak processes by Fermi (1934), the well-known (V–A) theory.

We notice that in spite of the fact that Ω and χ enter symmetrically in the Lagrangian density, they have different dimensionless coupling strengths ω and ξ. In the pure Brans-Dicke theory ($\chi \to 0$, $\xi \to 0$) it is possible, from considerations in celestial mechanics, to set a limit on permissible values of ω; for example, in the calculation of the perihelion rotation of a planetary orbit, the observational data requires (Brans & Dicke, 1961) $\omega \geqslant 6$. Similar requirements can be put on ξ from microphysics.

In the following section we shall see that the correspondence principle leads to different functional behaviour for the scalars, and that the Yukawa behaviour of χ leads us to consider this function as the *strong gravity* scalar field. Similarly, the Newtonian functional behaviour for the Ω field leads to the consideration of this field as the *weak* gravity scalar.

3. *The Classical Limits of the Unified Theory*

In view of the enormous difference in coupling strengths between strong and weak gravity (nearly forty orders of magnitude!), it is physically meaningful to discuss strong and weak gravities separately as a first-order approximation. Then, once the leading order functions Ω and χ in the asymptotic expansion of the coupling strength ϕ of gravity have been determined, we can proceed to study the implications of the *full* theory.

3.1. *The $\xi \to 0$, $\chi \to 0$ Limit (Scalar-Tensor Theory of Weak Gravity)*

In general relativity the equation of motion of a point particle without spin, moving in a weak gravitational field, may be obtained from a variational principle,

$$0 = \delta \int m(g_{\mu\nu} U^\mu U^\nu)^{1/2} \, ds$$

namely, one obtains the geodesic equation

$$\frac{d}{ds}(mU_\mu) - \tfrac{1}{2} m \partial_\mu g_{\nu\sigma} U^\sigma U^\nu = 0 \tag{3.1.1}$$

In this weak gravity field, we have (Brans & Dicke, 1961)

$$m = m_0 f(x) \tag{3.1.2}$$

so that equation (3.1.1) is modified to

$$\frac{d}{ds}(mU_\mu) - \tfrac{1}{2} m \partial_\mu g_{\nu\sigma} U^\sigma U^\nu - \partial_\mu m = 0 \tag{3.1.3}$$

We notice that equation (3.1.3) does not coincide with the standard equation for a geodesic of the geometry. However, the effect of a Weyl transformation

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = f^2 g_{\mu\nu}, \qquad f = \exp(\Lambda)$$

is given by

$$d\bar{s}^2 = f^2 ds \qquad \bar{U}^\mu = \frac{1}{f} U^\mu$$

Hence equation (3.1.3) becomes

$$\frac{d}{ds}(m_0 U_\mu) - \tfrac{1}{2} m_0 \partial_\mu \bar{g}_{\nu\sigma} \bar{U}^\sigma \bar{U}^\nu = 0 \tag{3.1.4}$$

and the particle moves on a geodesic of the rescaled geometry. Now, the field is weak in the sense that if we write

$$\bar{g}_{\mu\nu} = \bar{g}_{\mu\nu}^{(\mu)} + \bar{\gamma}_{\mu\nu}, \qquad g_{\mu\nu}^{(\mu)} = dg\,(1, -1, -1, -1) \tag{3.1.5}$$

and if the potentials $\gamma_{\mu\nu}$ are dominated (except near the origin) by $\gamma_{\mu\nu}$ as compared with higher powers, then we say the field is weak. We may further assume that the point particle is moving slowly, in such cases we may approximate equation (3.1.4) as

$$\frac{d^2}{dt^2} \mathrm{x} = -\frac{c^2}{2} \partial_{\mathrm{x}} \bar{\gamma}_{00} \tag{3.1.6}$$

Then, provided we identify $(c^2/2)\,\bar{\gamma}_{00}$ with the classical r^{-1} potential, we recover in equation (3.1.5) the Newtonian limit of weak gravity.

3.2. *The* $\omega \to 0$, $\Omega \to 0$ *Limit (Theory of Strong Gravity)*

We first remark that equation (3.1.6) provides a self-consistent test for our approximation, since the classical limit provides a potential which fulfills the requirement that the $\bar{\gamma}_{\mu\nu}$'s dominate the deviations of the Minkowskian metric to first order. We should point out that no mention is made of the numerical magnitude of the coupling constant, since what we are after is the asymptotic behaviour of the scalar functions.

In the strong gravity limit, we also consider a point particle moving slowly along a geodesic of the rescaled geometry.

In writing the geodesic equations under the assumption (3.1.5) we consider the perturbations $\bar{h}_{00}$ of the flat metric $\bar{g}^{(\mu)}_{\mu\nu}$,

$$\bar{g}_{\mu\nu} = \bar{g}^{(\mu)}_{\mu\nu} + \bar{h}_{\mu\nu} \tag{3.2.1}$$

to be due entirely to the gauge function Λ, hence identifying the gauge function with the only scalar function, namely, the strong gravity scalar. It should be observed that such identification was precisely Dicke's viewpoint (in weak gravity) in his later work on Mach's principle and invariance under transformation of units (Dicke, 1962).

We are therefore led, as in the weak gravity limit, to the classical equation satisfied by the perturbation $\bar{h}_{00}$ (cf. equation (3.2.1) of the Minkowski metric entirely due to the strong gravity scalar field

$$\frac{d^2}{dt^2}\mathrm{x} = -\frac{c^2}{2}\,\partial_{\mathrm{x}}\bar{h}_{00}$$

Hence, if V represents the classical potential corresponding to strong gravity (Yukawa's potential), we find

$$\bar{g}_{00} = 1 + 2V/c^2$$

or, equivalently

$$\bar{g}_{00} \equiv \exp(2\Lambda)g_{00} = 1 + 2V/c^2 \tag{3.2.2}$$

In view of the fact that the main assumption of strong gravity is that in the classical limit

$$g_{00} = 1$$

we find the gauge function to be given by

$$\Lambda = V/c^2 = \exp(-\mu r)/c^2 r$$

where μ is the nuclear range.

Therefore, in the strong gravity limit, we make the Dicke-type identification of the Weyl gauge function Λ with the strong gravity scalar χ (which is indeed interpreted as a potential from equation (3.2.2)).

Finally, to conclude this section we may return to the asymptotic expansion (2.1) to find

$$\phi^2 = \Omega^2 + \chi^2 \tag{3.2.3}$$

where the field functions are given by

$$\Omega^2 = A/r \qquad (3.2.4)$$

$$\chi^2 = B\exp(-\mu r)/r \qquad (3.2.4)$$

4. *Physical Implications of the Unified Theory*

(a) *The Einstein Theory.* A special case of the $\xi \to 0$ limit is of interest here, namely, if we further approximate Ω^2 by the constant G_N^{-1} we are led to the Einstein field equations.

(b) *The Brans–Dicke Theory.* This theory is recovered in the $\xi \to 0$ limit, while retaining a variable Ω^2.

(c) *The C–H Theory.* In the $\omega \to 0$ limit we may approximate χ^2 by the constant G_{ST}^{-1} and recover the C–H model (cf. appendix A).

If the full equations for the metric are written (explicitly), we find

$$(\Omega^2 + \chi^2)G_{\mu\nu} = T_{\mu\nu} + S_{\mu\nu}(\Omega, \chi) \qquad (4.1)$$

For short-range forces

$$\chi^2 \gg \Omega^2 \qquad \text{(numerically)} \qquad (4.2)$$

thus the strong gravity scalar χ dominates the physics at the microscopic level.

At the same time, the same field equation (4.1) serves in the discussion of ordinary gravitational problems (planetary physics, cosmology, but not cosmogony), for such problems are clearly of long-range order, hence

$$\Omega^2 \gg \chi^2$$

since the Yukawa potential cuts off around one fermi.

5. *Extension Into Gauge Theories of Fundamental Forces*

5.1. *The Freund Theory*

By gauging the Weyl invariance of our Lagrangian density in the long-range limit, one is clearly led to a non-linear set of coupled equations for A_μ interpreted as the photon field, the $g_{\mu\nu}$ and the single scalar (essentially our) Ω.

Spontaneous breakdown of gauge invariance (the Higgs–Kibble mechanism (Kibble, 1967; Higgs, 1964)) led Freund (1974) to a violation of C-invariance since the crossed terms in the piece

$$\mathscr{L} = (\partial_\mu - eA_\mu)\,\phi\,(\partial_\nu - eA_\nu)\,\phi g^{\mu\nu} \qquad (5.1.1)$$

of the total Lagrangian violate C-symmetry (A_μ is odd under C-exchanges, $A_\mu^c = -A_\mu$). Hence the presence of the terms $eA_\mu\phi\partial_\nu\phi g^{\mu\nu}$ produces the (spontaneously broken) discrete symmetry violation, C as well as CP.

The effect, however, is far too small to account for the observed Fitch-Cronin CP violating effect in $K_l \to \pi^0 \pi^0$ (Christensen *et al.*, 1964). However, Freund remarks that if one replaces weak gravity by strong gravity, the correct order of magnitude is obtained.

From the point of view of our theory, after gauging Weyl invariance and spontaneously breaking the symmetry of the (quantised) theory, we are led through the Higgs-Kibble mechanism to a *massive* photon field, indicating that the long-range limit is no longer suitable, instead we have

$$\chi^2 \gg \Omega^2$$

Thereby providing us with an understanding of the *replacement* of weak gravity by strong gravity, which was done phenomenologically by Freund, as well as by ourselves in the C-H model (Chela-Flores & Herrera, 1974).

5.2. *An Interesting Numerical Coincidence*

In a recent paper of Georgi, Quinn & Weinberg (1974), renormalisation effects are studied which make strong interactions strong, in gauge theories of strong, electromagnetic and weak interactions. The above-mentioned authors are led to *super-heavy gauge bosons* arising in the spontaneous breakdown of symmetries in observed interactions. (We refer to such bosons as the Z-particles.) They find

$$m_Z = 10^{18\pm1}\,\mathrm{GeV} \qquad (5.2.1)$$

Although the improved gauge theory of electromagnetic and gravitational interactions does not include weak interactions, we observe that if we had (erroneously) taken the long-range limit

$$\chi \ll \Omega$$

in a theory which is clearly of short-range nature, we would have found

$$m_Z = \Omega \cong 10^{19}\,\mathrm{GeV} \qquad (5.2.1)$$

(cf. Appendix B), which is clearly prevented by taking the correct short-range limit.

6. *Conclusions*

We have learnt that gauging the scalar-scalar-tensor theory may be a useful way to probe at a deeper level the role played by gravitational forces in elementary processes. The semi-classical results of the C-H model are seen in a different light within the context of the two gravities and, particularly, the improved unified theory of electromagnetism and gravitation gains in simplicity and inner consistency.

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Appendix A

Point (c) of Section 4 needs some clarification since, in the C–H theory, *both* the cosmological constant and the strong gravity coupling strength were rescaled.

The relevant observation here is that the bounds on highly dense matter (hadronic matter) were obtained for a completely degenerate relativistic gas of fermions; so, in order to understand properly the coupling of fermions to curved space, we must extend Einstein's equations (without a cosmological constant) to include *torsion* as explained, for instance, by Trautman (1973).

Recently, however, Prasanna (1973) has solved explicitly the Einstein–Cartan field equations and found that the assumption of a spherically symmetric distribution of spin-1/2 particles puts a natural restriction on the spins of the particles, namely that they all have only the radial component different from zero. The density of spins appears in the field equations (cf. Prasanna's equations (4.10)) as a term which we observe to be equivalent to a 'cosmological' constant; the constant, in fact, appears as being due to the total spin conservation. It is, further, of the right magnitude since the density of spin constant is proportional to the gravitational scalar coupling which in the C–H theory is microscopic ($\chi \gg \Omega$).

Appendix B

Following Freund (1974), when we gauge the Weyl symmetry, we must consistently replace derivatives by covariant derivatives. In particular the Riemann scalar must change accordingly,

$$\bar{R} = \bar{R}[\partial_\mu g_{\mu\lambda} \to \nabla_\mu g_{\mu\lambda}], \qquad \nabla_\mu\text{: covariant derivative} \tag{B.1}$$

More explicitly, the affine connections transform as

$$\Gamma^\mu{}_{\nu\lambda} \to \bar{\Gamma}^\mu{}_{\nu\lambda} = \tfrac{1}{2} g^{\mu\rho}[(\partial_\lambda + 2eA_\lambda)g_{\rho\nu} + (\partial_\nu + 2eA_\nu)g_{\lambda\rho} - (\partial_\rho + 2eA_\rho)g_{\nu\lambda}]$$

Hence, in terms of the old Riemann scalar, we have the transformation law,

$$\bar{R} = R + 6e\partial_\mu A^\mu + 6e^2 A_\nu A^\mu + 6eA^\mu \Gamma_\mu{}^\beta{}_\beta \tag{B.2}$$

We notice from the way ϕ^2 and R are coupled, that once there is spontaneous breakdown of symmetry the scale of the mass of the massive gauge boson will be determined by χ in the short-range limit and, therefore, the mass of the gauge field will be of the order of the proton mass and *not* of the superheavy type found in Georgi *et al.* (1974).

Finally, we notice that the crossed term $eA_\mu \phi \partial_\nu \phi g^{\mu\nu}$ of the total Lagrangian density in equation (5.1.1) will also contribute to the total mass, but this does not change the conclusions.

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